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THE OCTAGONAL GIRDER FOUR COLUMN SPACE FRAME

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THE OCTAGONAL GIRDER FOUR COLUMN SPACE FRAME

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SYNOPSIS

This paper presents the analysis of a space frame consisting of a closed octagonal ring supported rigidly on four columns. A solution was obtained in closed form for three types of loading:

a.) Uniformly distributed vertical load,
b.) Concentrated horizontal (or shear) load and
c.) Linearly varying antisymmetrical (moment) loading.

In the derivation it has been assumed that the girder is infinitely rigid in its plane; this stipulation is physically very nearly correct.

The calculations are extended to cover the fixed and hinged conditions at the bottom of the supporting columns. The moments at the segmental points and at the column-ring junctions as well as the end reactions are expressed explicitly and presented in tabular form; these expressions appear as functions of only the load, the ratio of bending to torsional stiffness of the ring (k) and the ratio of relative bending stiffness of the ring and the columns (n).

In addition to an extended analysis, a modified analysis is presented which results in material simplifications of the functions and computations involved in the moment expressions. Both types of analyses are accompanied by numerical examples.

The general method of analysis is adaptable to ring girders of different shapes or a greater number of supporting columns.

NOMENCLATURE

M	Bending moment, as defined
T	Torsional moment, as defined
V	Shear or vertical force, as defined
θ	Rotation of structural element
Δ	Translation or deflection of structural element
w	Vertical loading per unit of length, as defined
P	Extraneous horizontal load acting in the plane of the girder
M_o	Extraneous moment acting perpendicular to the plane of the girder
U	Total strain-energy in loaded structure

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E	Modulus of elasticity	of horizontal girder
I	Moment of inertia	
G	Shear modulus	
J	Torsional moment of inertia	
b	Width	
d	Depth	
a	One-half of segmental length	
E_c	Modulus of elasticity	of supporting square columns
I_c	Moment of inertia	
c	Side length of cross-section	
h	Height	

$k = \frac{EI}{GJ}$ Ratio of bending to torsional stiffness of ring

$n = \frac{hI}{aI_c}$ Ratio of relative bending stiffness of ring and columns

Note: Double subscripts denote generalized moments in, or rotation of, a member indicated by the subscript.

In case of a concurrent subscript and superscript, the subscript denotes the location and the superscript the direction of the moment or the sense of rotation in question.

INTRODUCTION

The majority of structures in existence are space structures subjected to loads in three dimensions. Curved or segmental frames are frequently employed in the form of supporting structures for chemical equipment, in ship construction, for bins or water towers, curved balcony frames, arches, horizontally curved bridges or components of piping systems.

For convenience and simplicity, most of these structures are reduced in design to planar considerations. This simplification, however, leaves a number of questions pertaining to the load-carrying capacity of these structures unanswered. It becomes, therefore, necessary to use a higher factor of safety, so that the inherent strength of spatial frames is not fully utilized to its full degree.

As an example, present refinery tower supports normally consist of a rectangular slab with a central octagonal cut-out supported on four columns. The design of this slab poses a certain difficulty, since it is doubtful whether it should be designed according to the slab or beam specifications of the Code. In addition, it was long suspected that sizable savings in material could be achieved if a segmental octagonal ring were used as a replacement of the current slab design for these pedestals.

The analysis of these structures is, by necessity, quite involved. A systematic, although somewhat brief study of horizontally curved beams is given by Timoshenko (11). In addition, many investigators (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16) have studied various aspects of the problem of curved beams having different modes of end fixity and being loaded normal to their plane of curvature. Spatially curved piping systems, a special group of the spatial frame problems; have also been subjected to numerous analytical studies (17, 18, 19, 20, 21, 22).

To the authors' knowledge, however, no study exists analyzing the stresses in space-frames with horizontally curved members, subject to loading in, or normal to, their plane of curvature. This study was carried out to initiate an analysis of structures of this nature, and to foster interest for further development in this field.

The structures analyzed herein are space frames consisting of a closed octagonal ring, supported rigidly on 4 evenly spaced columns, which are either hinged or fixed to their foundations. The method of solution presented is easily extendable to ring structures of a different shape, or to an arbitrary number of supporting columns.

EXTENDED ANALYSIS

A. Designation of Forces and Method of Attack

The general shape of this structure is shown in Figure 1. The octagonal ring has a rectangular cross-section, the width of which is denoted by b and the depth by d. The columns are assumed to have a square cross-section (length of sides equal to c), although this is not a mandatory assumption from the viewpoint of analysis.

The solution, in most cases, is based upon Castigliano's theorem or the Principle of Least Work (23). A curvilinear orthogonal coordinate system is employed, whose positive x-axis is directed in the forward sense of travel from A to B to C to D (see Figure 1.) The positive y-axis is obtained by a 90 degree clockwise rotation of the x-axis, (i.e. the y-axis always points toward the outside of the ring). This coordinate system suffers abrupt directional changes at the corner-points of the octagonal ring.

A vector notation, using the right-hand screw rule, is used to designate the moments. Bending and twisting moments in the girder are denoted by M and T, and are taken positive if their vector points in the direction of the positive y and x-axis, respectively. Vertical forces are positive if their vector points downwards. Moment vectors are designated by an arrow with a front and back head, whereas forces have the usual vector sign. A double subscript on moments depicts a generalized moment over a member of the closed ring; a concurrent subscript and superscript indicates a specific moment, whose location is given by the subscript and orientation by the superscript. Moments at a given location in the ring will be understood to act on the right side (far side in the sense of travel) of a given section.

Bending moments in the supporting columns will be taken positive if their vector is oriented parallel to the positive sense of the x or y axes. These moments will again be affixed by a subscript, indicating their location, and a superscript, indicating their orientation in the columns.

B. Uniformly Distributed Vertical Load

Under the loading arising from the weight of the supported equipment, the distribution of applied and reacting forces and moments is symmetrical about both the A-C and B-D axes; hence no torque can exist in the structure at these points. The bending moment in the ring at the supports, however, is indeterminate.

By symmetry it suffices to take only one quadrant of the structure under scrutiny. This is shown in Figure 2, where M' denotes the redundant bending moment and T' indicates the balancing torque. The quadrant is assumed to be free to rotate at its supports, and will be in equilibrium only if

$$M' = 3/2\ wa^2 - T' \qquad (1)$$

The ensuing analysis is given in its full extent for the sake of clarity, and to eliminate the tedium of repetitive details throughout the rest of the paper.

The moment expressions for the members of the quadrant take the following form:

MEMBER A-1

1 $$M_{A-1} = -2wax + \frac{wx^2}{2} + M' \quad ; \quad T_{A-1} = -T'$$

MEMBER 1-2

2 $$M_{1-2} = \frac{-3}{2\sqrt{2}} wa^2 - wax + \frac{wx^2}{2} + \frac{M'}{\sqrt{2}} - \frac{T'}{\sqrt{2}}$$ 3 $$T_{1-2} = \frac{3}{2\sqrt{2}} wa^2 - \frac{M'}{\sqrt{2}} - \frac{T'}{\sqrt{2}}$$

MEMBER 2-B

4 $$M_{2-B} = wax + \frac{wx^2}{2} - T' \quad ; \quad T_{2-B} = \frac{3}{2} wa^2 - M'$$

By Castiliagno's theorem, the torsional rotation at points A or B, under the external forces shown in Figure 2, are obtained from the equation:

5 $$\theta_A^T = \frac{\partial U}{\partial T'} = -\frac{\partial}{\partial T'}\left[\sum_{\text{MEMBER A-1}}^{\text{MEMBER 2-B}} \int_0^{\ell} \left(\frac{M^2}{2EI} + \frac{T^2}{2GJ}\right) dx\right] =$$

$$= -\sum_{\text{MEMBER A-1}}^{\text{MEMBER 2-B}} \left[\frac{1}{EI} \int_0^{\ell} \left(M\frac{\partial M}{\partial T'} + kT\frac{\partial T}{\partial T'}\right) dx\right] = \left[\frac{2-\sqrt{2}}{3} wa^2 - (3+k)T'\right]\frac{a}{EI}$$

where

5A $k = \frac{EI}{GJ}$ and ℓ is the length of the member in question.

Further analysis must take into account the mode of fixity of the columns.

1. Columns Hinged at the Base

By symmetry, the bending moments in the ring on two sides of any support are equal and opposite, so that the columns are subject to bending only in the diametral normal planes of the ring (planes A-A′ - C-C′ and B-B′ - D-D′ in Figure 1), under the action of a moment equal to $2T'$ applied at their top. Hence, for hinged-base columns, the top rotation is given by

$$\theta_{\text{COL A TOP}}^{X-X} = \frac{2T'h}{3E_c I_c}$$

Since the torsional rotation of the ring at the supports must equal the bending rotation of the top of the columns, one has to satisfy the condition

$$\theta_A^T = \theta_{\text{COL. A TOP}}^{x-x} \tag{7}$$

Assuming now that $E_c = E$ (this assumption is tacitly understood throughout the rest of this paper), and denoting the relative stiffness of the ring and column by

$$n = \frac{h}{a}\frac{I}{I_c} \tag{8}$$

one obtains, by substitution of Eqs. (2) and (3) into Eq. (4)

$$T' = \beta_1 wa^2 \tag{9}$$

where

$$\beta_1 = \frac{0.5858}{2n+3k+9} \tag{10}$$

Inserting Eq. (5) into Eq. (1) there obtains

$$M' = (1.5 - \beta_1)\, wa^2 \tag{11}$$

In possession of M' and T', it is possible to derive explicit expressions for the moments at any point in the ring and columns from the moment equations given at the outset. These moments are given in Table 1. The twisting moments over member 1-2 are equal to zero, since symmetry about the axis E - E' excludes the presence of a torque at E.

β_1 vanishes when either $n \to \infty$ or $k \to \infty$; in these cases both twisting moments in the ring and bending moments in the columns also tend to zero, and the solution reverts to a statically determinate one.

A simplification from the standpoint of design can be achieved in the following way:

For a rectangular cross section (width b, depth d) $I = 0.08333\ bd^3$. From elasticity (24), $G = E/2(1+\nu)$, and assuming $\nu = 0.15$ for reinforced concrete), there results $G = 0.4348E$.

Furthermore, for a cross section with a depth to width ratio varying from 0.33 to 3.0, an excellent approximation is obtained if one assumes (25)

$$J = \frac{0.286}{b^2+d^2}\, b^3 d^3 \tag{12}$$

With these expressions the equation for k can be rearranged into the following form

$$k = \frac{EI}{GJ} = 0.6702\left(1 + \frac{d^2}{b^2}\right) \tag{13}$$

this formula being valid over the range $\frac{1}{3} < \frac{d}{b} < 3$.

The values of d/b represent tangible design dimensions. It has to be borne in mind, however, that for extremely flat or deep cross sections the expression for J will not hold; in these cases recourse has to be made to Timoshenko (25). Furthermore, for steel structures ν is equal to 0.30, which will also alter the character of Eq. (7).

2. Columns Fixed at the Base

The approach is similar to the previous case, except that Eq. (3) modifies to

$$\theta^{x-x}_{COL\,A_{TOP}} = \frac{T'h}{2E_cI_c} \tag{14}$$

Thus, combining Eq. (2), (8), and (4), there results

$$T' = \beta_2 w a^2 \tag{15}$$

where

$$\beta_2 = \frac{0.3905}{n + 2k + 6} \tag{16}$$

Substitution of Eq. (9) into Eq. (1) gives

$$M' = (1.5 - \beta_2)\, w a^2 \tag{17}$$

The bending and twisting moments for a uniformly loaded octagonal pedestal on four fixed end columns are shown in Table 2. Table 2 is, in general, analogous to Table 1, except that in this table β_2 takes over the role of the function β_1 in Table 1. For both cases (hinged or fixed end columns) maximum bending moments and torques are obtained at point A in the octagonal ring.

Comparing the solutions for the hinged and fixed end columns, it appears that $\beta_1 = \beta_2$ when n = 0 or n = ∞; for these cases, therefore, the solutions coincide. When $0 < n < \infty$, $\beta_1 < \beta_2$, so that greater maximum bending moments but smaller maximum torques are developed in the ring for the hinged-column pedestal. The bending moments in the columns are also smaller in this case as compared to the pedestal with fixed columns.

TABLE 1

Uniformly Distributed Vertical Load, w. Hinged Columns

OCTAGONAL RING

Location and Direction	Bending Moments	Twisting Moments
At A in direction of 1	$M_A^1 = (1.5 - \beta_1)wa^2$	$T_A^1 = -\beta_1\ wa^2$
At 1 in direction of A	$M_1^A = -\beta_1\ wa^2$	$T_1^A = -\ "\ -$
At 1 in direction of E	$M_1^E = -\sqrt{2}\,\beta_1\ wa^2$	$T_1^E = 0$
At E	$M_E = -(0.5 + \sqrt{2}\,\beta_1)wa^2$	$T_E = 0$
At 2 in direction of E	$M_2^E = -\sqrt{2}\,\beta_1\ wa^2$	$T_2^E = 0$
At 2 in direction of B	$M_2^B = -\beta_1\ wa^2$	$T_2^B = \beta_1\ wa^2$
At B in direction of 2	$M_B^2 = (1.5 - \beta_1)\ wa^2$	$T_B^2 = -\ "\ -$

COLUMNS

Bending Moments at Column Tops	Bending Moments at Column Bases
$M^{x-x}_{\text{Col.A top}} = 2\beta_1\ wa^2$	$M^{x-x}_{\text{Col.A base}} = 0$
$M^{y-y}_{\text{Col.A top}} = 0$	$M^{y-y}_{\text{Col.A base}} = 0$
$M^{x-x}_{\text{Col.B top}} = 2\beta_1\ wa^2$	$M^{x-x}_{\text{Col.B base}} = 0$
$M^{y-y}_{\text{Col.B top}} = 0$	$M^{y-y}_{\text{Col.B base}} = 0$

Vertical Loads	Horizontal Reactions at Base
$V_{\text{Col.A}} = 4aw$	$H^{x-x}_{\text{Col.A}} = 0$
$V_{\text{Col.A}} = 4aw$	$H^{y-y}_{\text{Col.A}} = \frac{2\beta_1}{h}\ wa^2$
	$H^{x-x}_{\text{Col.B}} = 0$
	$H^{y-y}_{\text{Col.B}} = \frac{2\beta_1}{h}\ wa^2$

C. Horizontal Shear Loading

Wind pressures exerted on the supported superstructure are transmitted to the frame as a concurrent horizontal shear and overturning moment. Although the exact distribution of these shear forces is not known, it is a plausible approach to unify them into a single resultant, and then study the action of the frame under the attack of a concentrated horizontal force located in the plane of the ring.

In the following analysis the ring is assumed to be infinitely rigid in its plane; hence no bending moments can develop in this plane. This assumption is physically very nearly correct and permits great simplifications in the mode of attack of the problem.

The structure may be exposed to a wind attack from any arbitrary direction. Greatest bending moments and vertical shears arise, however, when the wind load is parallel to the planes defined by a set of diametrically opposite columns, (e.g. plane A-A' - C-C' in Fig. 1); greatest twisting moments occur when the wind load attacks on vertical planes oriented at 45° to the previous direction. The analysis scrutinized in detail herein refers to the lateral loading directed in the plane A-A' - C-C'. The functions for wind loading at 45° to this direction can be obtained by superimposing the component effects of two loads of 0.7071P magnitude, attacking on the perpendicular planes of A-A' - C-C' and B-B' - D-D', respectively.

As an auxiliary part of the solution, a study of the octagonal ring under the action of two equal moments $\bar{T}$ applied at A and C, and two equal moments $\bar{M}$ applied at B and D has to be made, these four moments being undirectional, as shown in Figure 3. The geometry of structure and loading give rise to the following limitations:

From antisymmetry about axis B-D:

$$V_B = V_D = M_B = M_D = \theta_B^T = \theta_D^T = 0 \; ; \; V_A = -V_C \tag{18}$$

From Symmetry about axis A-C:

$$T_A = T_C = \theta_A^M = \theta_C^M = 0 \tag{19}$$

Symmetry also permits to limit the study to the semi-octagonal ring extending from A to C, presented in Fig. 4. This figure shows, in addition to the external moments $\bar{T}$ and $\bar{M}$, a moment M'', which represents the redundant bending moment existing at supports A and C.

Employing the Theorem of Least Work, the condition

$$\theta_A^M = \frac{\partial U}{\partial M''} = 0 \tag{20}$$

provides the expression for the redundant bending moment. One obtains:

$$M'' = \frac{0.3536k + 0.3018}{k+1}\bar{M} + \frac{0.6036k + 0.05178}{k+1}\bar{T} \tag{21}$$

This expression enables one to obtain the torsional rotation over support A and the bending rotation of the ring over support B solely in terms of the applied external moments $\bar{M}$ and $\bar{T}$:

TABLE 2

Uniformly Distributed Vertical Load, w. Fixed Columns

OCTAGONAL RING	
Bending Moments	**Twisting Moments**
$M_A^1 = (1.5 - \beta_2) wa^2$	$T_A^1 = -\beta_2 wa^2$
$M_1^A = -\beta_2 wa^2$	$T_1^A = -\,"\,-$
$M_1^E = -\sqrt{2}\,\beta_2 wa^2$	$T_1^E = 0$
$M_E = -(0.5 + \sqrt{2}\,\beta_2) wa^2$	$T_E = 0$
$M_2^E = -\sqrt{2}\,\beta_2 wa^2$	$T_2^E = 0$
$M_2^B = -\beta_2 wa^2$	$T_2^B = \beta_2 wa^2$
$M_B^2 = (1.5 - \beta_2) wa^2$	$T_B^2 = -\,"\,-$
COLUMNS	
Bending Moments at Column Tops	**Bending Moments at Column Bases**
$M^{x-x}_{\text{col.A top}} = 2\,\beta_2 wa^2$	$M^{x-x}_{\text{col.A base}} = -\beta_2 wa^2$
$M^{y-y}_{\text{col.A top}} = 0$	$M^{y-y}_{\text{col.A base}} = 0$
$M^{x-x}_{\text{col.B top}} = 2\,\beta_2 wa^2$	$M^{x-x}_{\text{col.B base}} = -\beta_2 wa^2$
$M^{y-y}_{\text{col.B top}} = 0$	$M^{y-y}_{\text{col.B base}} = 0$
Vertical Loads	**Horizontal Reactions at Base**
$V_{\text{col.A}} = 4\,aw$	$H^{x-x}_{\text{col.A}} = 0$
$V_{\text{col.B}} = 4\,aw$	$H^{y-y}_{\text{col.A}} = \frac{3\beta_2}{h} wa^2$
	$H^{x-x}_{\text{col.B}} = 0$
	$H^{y-y}_{\text{col.B}} = \frac{3\beta_2}{h} wa^2$

22 $$\theta_A^T = \frac{\partial U}{\partial T} = \frac{a}{EI}\left[\zeta_1 \bar{M} + \zeta_2 \bar{T}\right]$$

23 $$\theta_B^M = \frac{\partial U}{\partial M} = \frac{a}{EI}\left[\zeta_3 \bar{M} + \zeta_1 \bar{T}\right]$$

where ζ_1, ζ_2 and ζ_3 can be represented, to a four digit accuracy, by the following functions:

24 $$\zeta_1 = \frac{-0.06066k^2 + 0.001945k - 0.05167}{k+1}$$

25 $$\zeta_2 = \frac{0.5429k^2 + 1.8644k + 0.1037}{k+1}$$

26 $$\zeta_3 = \frac{0.08579k^2 + 0.3969k + 0.3004}{k+1}$$

Note that Eqs. (11) satisfy the conditions of Maxwell's Law of Reciprocal Relations.

Further treatment must again be subdivided according to the mode of fixity of the columns.

1. Columns Hinged at the Base

Fixing the columns momentarily at the top and treating the octagonal ring as a rigid beam, every column deflects through a distance Δ under the action of the horizontal load, as shown in Fig. 5. The resulting moment of the top of each column is

27 $$M_{COL.TOP} = \frac{3EI_c}{h^2}\Delta = \frac{Ph}{4}$$

If the column-ring joints are released, the displacements increase to Δ_1, while the column-ring junctions rotate through an angle of θ_A^T at A and θ_B^M at B. Hence the moments in the columns become

28 $$M_{COL\ ATOP}^{X-X} = \frac{3EI_c}{h}\left(\frac{\Delta_1}{h} - \theta_A^T\right)$$

29 $$M^{y-y}_{COL.BTOP} = \frac{3EI_c}{h}\left(\frac{\Delta_1}{h} - \theta^M_B\right)$$

Eliminating Δ_1 from these equations, there results

30 $$\frac{h}{3EI_c} M^{x-x}_{COL.ATOP} + \theta^T_A = \frac{h}{3EI_c} M^{y-y}_{COL.BTOP} + \theta^M_B$$

Combining this expression with the condition

31 $$M^{x-x}_{COL.ATOP} + M^{y-y}_{COL.BTOP} = \frac{Ph}{2}$$

one arrives at the following relations for the column-top moments:

32 $$M^{x-x}_{COL.ATOP} = \frac{Ph}{4} + \frac{3EI_c}{2h}\left(\theta^M_B - \theta^T_A\right)$$

33 $$M^{y-y}_{COL.BTOP} = \frac{Ph}{4} + \frac{3EI_c}{2h}\left(\theta^T_A - \theta^M_B\right)$$

The condition that moments in the ring and columns be equal at their junction demands that

34 $$M^{x-x}_{COL.ATOP} = \bar{T} \quad ; \quad M^{y-y}_{COL.BTOP} = \bar{M}$$

where $\bar{T}$ and $\bar{M}$ are the moments under whose action the ring was analyzed previously. Using this condition, substituting Eq. (11) into Eq. (12) and solving the resulting simultaneous equations for the column-top moments yields the expressions:

35 $$M^{x-x}_{COL.ATOP} = \frac{n + 3(\zeta_3 - \zeta_1)}{2[2n + 3(\zeta_2 + \zeta_3 - 2\zeta_1)]} Ph$$

36 $$M^{y-y}_{COL.BTOP} = \frac{n + 3(\zeta_2 - \zeta_1)}{2[2n + 3(\zeta_2 + \zeta_3 - 2\zeta_1)]} Ph$$

Finally, substitution of the ζ functions into these expressions, and observing that

37 $$T_A^1 = -\frac{1}{2} M_{COL.A\,TOP}^{x-x}, \qquad M_B^2 = \frac{1}{2} M_{COL.B\,TOP}^{y-y},$$

result

38 $$T_A^1 = (\gamma_1 - 0.25)Ph$$

39 $$M_B^2 = \gamma_1 Ph$$

The moments and forces at any point in the structure can now be expressed in terms of the following two functions:

40 $$\gamma_1 = \frac{n(k+1)+1.8107k^2+5.587k+0.4660}{4[2n(k+1)+2.25k^2+6.772k+1.5221]}$$

41 $$\gamma_2 = \frac{n(1.9142k+0.7071)+1.8107k^2+4.709k+0.3906}{\text{SAME DENOMINATOR}}$$

The design quantities at the significant points in quadrant A-B of the pedestal are listed in Table 3. Over quadrant B-C the torques are equal, whereas bending moments are equal in magnitude but opposite in sign, as compared to those acting in quadrant A-B.

For the horizontal shear-loading, the location of maximum and minimum moments depends upon the values of k and n. However, within the range of variables normally encountered, the greatest negative bending moment and greatest absolute torque occurs at points A, whereas the maximum positive bending moment in the ring exists at junction B.

2. Columns Fixed at the Bottom

A rigid horizontal translation Δ of the ring induces the following moments at the column junctions

42 $$M_{COL.TOP} = \frac{6EI_c}{h^2}\Delta$$

If the junctions are released, the ensuing deflections and rotations will modify these moments as follows:

43 $$M_{COL.A\,TOP}^{x-x} = \frac{4EI_c}{h}\left(\frac{3\Delta_1}{2h} - \theta_A^T\right)$$

44 $$M^{y-y}_{COL.B\,TOP} = \frac{4EI_c}{h}\left(\frac{3\Delta_1}{2h} - \theta^M_B\right)$$

Elimination of Δ_1 gives rise to the identity:

45 $$M^{x-x}_{COL.A\,TOP} - M^{y-y}_{COL.B\,TOP} = \frac{4EI_c}{h}\left(\theta^M_B - \theta^T_A\right)$$

The governing condition for a concentrated horizontal force can now be given as

46 $$M^{x-x}_{COL.A\,TOP} + M^{y-y}_{COL.B\,TOP} = \frac{Ph}{4} - \frac{EI_c}{h}\left(\theta^M_B + \theta^T_A\right)$$

The latter two equations can be solved for the column-top moments. Further substitution of Eqs. (11) and the continuity condition with the solution yields the result

47 $$M^{x-x}_{COL.A\,TOP} = \frac{n[n+4(\zeta_3-\zeta_1)]}{4[2n^2+n(5\zeta_2+5\zeta_3-6\zeta_1)+8(\zeta_2\zeta_3-\zeta_1^2)]}\,Ph$$

48 $$M^{y-y}_{COL.B\,TOP} = \frac{n[n+4(\zeta_2-\zeta_1)]}{4[2n^2+n(5\zeta_2+5\zeta_3-6\zeta_1)+8(\zeta_2\zeta_3-\zeta_1^2)]}$$

Insertion of the ζ functions, in turn, will render these expressions to become

49 $$T^I_A = -\frac{1}{2}M^{x-x}_{COL.A\,TOP} = (\gamma_3 - \gamma_5)\,Ph$$

50 $$M^2_B = \frac{1}{2}M^{y-y}_{COL.B\,TOP} = \gamma_3\,Ph$$

so that all quantities of interest can be expressed with the aid of three auxiliary functions:

51

$$\gamma_3 = \frac{n[n(k+1)+2.414k^2+7.450k+0.6213]}{8[2n^2(k+1)+n(3.507k^2+11.295k+2.330)+0.3431k^3+2.662k^2+4.583k+0.2277]}$$

TABLE 3

Concentrated Horizontal Load, P. Hinged Columns

OCTAGONAL RING	
Bending Moments	**Twisting Moments**
$M_A^1 = -\gamma_2 Ph$	$T_A^1 = (\gamma_1 - 0.25) Ph$
$M_1^A = -(\gamma_2 - 0.10355) Ph$	$T_1^A = -$ " $-$
$M_1^E = 0.7071\ (\gamma_1 - \gamma_2 - 0.14645) Ph$	$T_1^E = 0.7071\ (\gamma_1 + \gamma_2 - 0.3536) Ph$
$M_E = 0.7071\ (\gamma_1 - \gamma_2) Ph$	$T_E = -$ " $-$
$M_2^E = 0.7071\ (\gamma_1 - \gamma_2 + 0.14645) Ph$	$T_2^E = -$ " $-$
$M_2^B = (\gamma_1 - 0.10355) Ph$	$T_2^B = (\gamma_2 - 0.25) Ph$
$M_B^2 = \gamma_1 Ph$	$T_B^2 = -$ " $-$

COLUMNS	
Bending Moments at Column Tops	**Bending Moments at Column Bases**
$M^{x-x}_{col.A\ top} = (0.5 - 2\ \gamma_1) Ph$	$M^{x-x}_{col.A\ base} = 0$
$M^{y-y}_{col.A\ top} = 0$	$M^{y-y}_{col.A\ base} = 0$
$M^{x-x}_{col.B\ top} = 0$	$M^{x-x}_{col.B\ base} = 0$
$M^{y-y}_{col.B\ top} = 2\ \gamma_1 Ph$	$M^{y-y}_{col.B\ base} = 0$
Vertical Loads	**Horizontal Reactions at Base**
$V_{col.A} = -\frac{0.2071}{a} Ph$	$H^{x-x}_{col.A} = 0$
$V_{col.B} = 0$	$H^{y-y}_{col.A} = (0.5 - 2\ \gamma_1) P$
	$H^{x-x}_{col.B} = -2\gamma_1 P$
	$H^{y-y}_{col.B} = 0$

52

$$\gamma_4 = \frac{n[n(1.9142k+0.7071)+2.414k^2+6.278k+0.5208]}{\text{SAME DENOMINATOR}}$$

53

$$\gamma_5 = \frac{n[2n(k+1)+3k^2+9.029k+2.029]}{\text{SAME DENOMINATOR}}$$

The moments and forces at the significant points in the fixed based pedestal under a horizontal force are given in Table 4. The location of moment maxima and minima are similar to those found for the hinged pedestal. The absolute maximum bending moment in the structure occurs at the base of column B. It is also of interest to note that the condition $k = \infty$ gives rise to equal moments in the four columns.

Although the analysis outlined above is concerned with moments due to the action of wind, it is equally well adaptable to any extraneous loading resulting in horizontal forces on the structure, as for instance earthquake effects or bomb blasts. It provides, furthermore, the results to be obtained from a sidesway analysis.

D. Moment Loading

The last part of the analysis is the study of the effect of an overturning moment arising from wind pressure exerted on the superstructure. This type of loading is equivalent to a linearly varying distributed load on the ring, assymmetrical about B-D axis, as shown in Figure 6. The greatest intensity of this load is $\bar{w}$, and it can be related to the overturning moment through the equation

54 $$M_o = 20.418\ \bar{w}a^2$$

The vertical plane of the overturning moment is taken to coincide with the horizontal force of Section B. Symmetry about the A-C axis facilitates a restriction of the analysis to one half of the structure. Some further conditions simplifying the analysis can be formulated in a manner identical to those given in Section B:

Due to antisymmetry:

55 $$V_B = V_D = M_B = M_D = \theta_B^T = \theta_D^T = 0 \ ; \quad V_A = -V_C$$

Due to symmetry:

56 $$T_A = T_C = \theta_A^M = \theta_C^M = 0$$

TABLE 4

Concentrated Horizontal Load, P. Fixed Columns

OCTAGONAL RING

Bending Moments	Twisting Moments
$B_A^1 = -\gamma_4 Ph$	$T_A^1 = (\gamma_3 - \gamma_5) Ph$
$B_1^A = -(\gamma_4 - 0.4142\gamma_5) Ph$	$T_1^A = -$ " $-$
$B_1^E = 0.7071\ (\gamma_3 - \gamma_4 - 0.5858\ \gamma_5) Ph$	$T_1^E = 0.7071\ (\gamma_3 + \gamma_4 - 1.4142\ \gamma_5) Ph$
$B_E = 0.7071\ (\gamma_3 - \gamma_4) Ph$	$T_E = -$ " $-$
$B_2^E = 0.7071\ (\gamma_3 - \gamma_4 + 0.5858\ \gamma_5) Ph$	$T_2^E = -$ " $-$
$B_2^B = (\gamma_3 - 0.4142\ \gamma_5) Ph$	$T_2^B = (\gamma_4 - \gamma_5) Ph$
$B_B^2 = \gamma_3 Ph$	$T_B^2 = -$ " $-$

COLUMNS

Bending Moments at Column Tops	Bending Moments at Column Bases
$M^{x-x}_{col.A\ top} = 2\ (\gamma_5 - \gamma_3) Ph$	$M^{x-x}_{col.A\ base} = (\gamma_3 + 0.5\gamma_5 - 0.25) Ph$
$M^{y-y}_{col.A\ top} = 0$	$M^{y-y}_{col.A\ base} = 0$
$M^{x-x}_{col.B\ top} = 0$	$M^{x-x}_{col.B\ base} = 0$
$M^{y-y}_{col.B\ top} = 2\ \gamma_3 Ph$	$M^{y-y}_{col.B\ base} = (-\gamma_3 + 1.5\ \gamma_5 - 0.25) Ph$

Vertical Loads	Horizontal Reactions at Base
$V_{col.A} = -\frac{0.8284}{a}\gamma_5 Ph$	$H^{x-x}_{col.A} = 0$
$V_{col.B} = 0$	$H^{y-y}_{col.A} = -(3\gamma_3 - 1.5\gamma_5 - 0.25) P$
	$H^{x-x}_{col.B} = -(3\gamma_3 - 1.5\gamma_5 + 0.25) P$
	$H^{y-y}_{col.B} = 0$

In addition to the external loads, there appears in Figure 6 a torque $\bar{\bar{T}}$ and a bending moment $\bar{\bar{M}}$, acting at the end points of the diameters A-C and B-D, respectively. These represent virtual couples needed for obtaining the rotation of the ring at these points.

Only the important features of the analysis are given below, without elaborating on the moment equations and derivatives.

The redundant bending moment may be obtained from the condition that

$$57 \qquad \theta_A^M = \frac{\partial U}{\partial \bar{\bar{M}}}\Bigg|_{\bar{\bar{M}}=\bar{\bar{T}}=0} = 0$$

This condition leads to

$$58 \qquad M'' = \frac{1.8333k + 1.6381}{k+1}\,\bar{w}a^2$$

The torsional rotation of the ring at supports A or C, under the loading shown, obtains from

$$59 \qquad \theta_A^T = \frac{\partial U}{\partial \bar{\bar{T}}}\Bigg|_{\bar{\bar{M}}=\bar{\bar{T}}=0} = k_1 \frac{\bar{w}a^3}{EI}$$

where

$$60 \qquad k_1 = \frac{-0.09069k^2 + 0.2570k - 0.08326}{k+1}$$

Similarly, the external loads initiate a bending rotation at supports B or D equal to

$$61 \qquad \theta_B^M = \frac{\partial U}{\partial \bar{\bar{M}}}\Bigg|_{\bar{\bar{M}}=\bar{\bar{T}}=0} = k_2 \frac{\bar{w}a^3}{EI}$$

where

$$62 \qquad k_2 = \frac{0.12826k^2 + 1.0692k + 0.9005}{k+1}$$

It can easily be demonstrated that the torsional rotation over supports B and D reduces to zero; this result could also be deduced from assymetry considerations.

The moments actually present in the structure under the type of loading shown in Figure 6 depend on the conditions of fixity at the base of the columns.

1. Columns Hinged at the Base

Every point of the octagonal ring is subject to bending and torsional rotations from two sources: the external load and the moments transmitted by the columns. The rotations at the points of support were obtained in Eqs. (17), (18) and Eqs. (11), respectively. Continuity requires that the rotation of the ring and column at their junction be equal; this condition stipulates that

At support A:

$$A: \quad k_1 \bar{w} a^2 - (\zeta_2 \bar{\bar{T}} + \zeta_1 \bar{\bar{M}}) = \frac{n}{3} \bar{\bar{T}} \tag{63}$$

At support B:

$$B: \quad k_2 \bar{w} a^2 - (\zeta_1 \bar{\bar{T}} + \zeta_3 \bar{\bar{M}}) = \frac{n}{3} \bar{\bar{M}} \tag{64}$$

where $\bar{\bar{T}}$ and $\bar{\bar{M}}$ represent the moments developing at the column to ring junctions. Solving for $\bar{\bar{T}}$ and $\bar{\bar{M}}$ there results

$$\bar{\bar{T}} = \frac{n k_1 + 3(k_1 \zeta_3 - k_2 \zeta_1)}{\frac{n^2}{3} + n(\zeta_2 + \zeta_3) + 3(\zeta_2 \zeta_3 - \zeta_1^2)} \bar{w} a^2 \tag{65}$$

$$\bar{\bar{M}} = \frac{n k_2 + 3(k_2 \zeta_2 - k_1 \zeta_1)}{\frac{n^2}{3} + n(\zeta_2 + \zeta_3) + 3(\zeta_2 \zeta_3 - \zeta_1^2)} \bar{w} a^2 \tag{66}$$

The continuity moments given by Eqs. (19) represent concurrently the moments applied to the top of the supporting columns. These moments give rise to a total horizontal reaction of

$$P_0 = \frac{2}{h} (\bar{\bar{T}} + \bar{\bar{M}}) \tag{67}$$

Equilibrium demands that the sum of horizontal reactions vanish; hence, for the moment loading shown, one must introduce a sidesway force acting in the sense analyzed in Section B, and equal in magnitude to Eq. (20). The moment corrections due to a sidesway can then be obtained from Table 3.

Eqs. (16), (19), substitution of the k and ζ functions and sidesway considerations permit a final formulation of the moments developing in the frame, with the aid of two functions:

$$\eta_1 = \frac{n^2(k+1)(0.008042k^2 + 0.02983k + 0.03013) + n(0.015168k^4 + 0.11082k^3 + 0.2803k^2 + 0.2813k + 0.04380) + 0.003105k^5 + 0.03560k^4 + 0.14475k^3 + 0.2643k^2 + 0.19394k + 0.009257}{n^3(k+1)^2 + n^2(k+1)(3.011k^2 + 10.170k + 1.9731) + n(2.508k^4 + 17.399k^3 + 33.92k^2 + 14.679k + 1.1786) + 0.4343k^5 + 4.676k^4 + 16.234k^3 + 20.025k^2 + 4.791k + 0.19498} \tag{68}$$

69 $$\eta_2 = \frac{n^3(k+1)(0.08979k+0.08022)+n^2(0.2744k^3+1.1656k^2+}{\text{SAME DENOMINATOR}}$$

$$\frac{+0.9962k+0.14022)+n(0.2328k^4+1.5784k^3+2.948k^2+}{\text{SAME DENOMINATOR}}$$

$$\frac{+1.0920k+0.07266)+0.04055k^5+0.4304k^4+1.4569k^3+}{\text{SAME DENOMINATOR}}$$

$$\frac{+1.7032k^2+0.2983k+0.01123}{\text{SAME DENOMINATOR}}$$

The forces and moments developing at various points in the frame subject to a moment loading are expressed in terms of η_1 and η_2 in Table 5, where recourse was made to the external moment by means of Eq. (15). Table 5 contains the critical points of quadrant A-B only, since the bending moments simply reverse their sense of action, whereas torques retain their signs over the quadrant B-C.

2. Columns Fixed at the Base

The continuity equations for the fixed base pedestal assume the following form:

At support A:

70 $$A : k_1 \bar{w}a^2 - (\gamma_2 \bar{\bar{T}} + \gamma_1 \bar{\bar{M}}) = \frac{n}{4} \bar{\bar{T}}$$

At support B:

71 $$B : k_2 \bar{w}a^2 - (\gamma_1 \bar{\bar{T}} + \gamma_3 \bar{\bar{M}}) = \frac{n}{4} \bar{\bar{M}}$$

The continuity moments can now be expressed as

72 $$\bar{\bar{T}} = \frac{nk_1 + 4(k_1\gamma_3 - k_2\gamma_1)}{\frac{n^2}{4} + n(\gamma_2+\gamma_3) + 4(\gamma_2\gamma_3 - \gamma_1^2)} \bar{w}a^2$$

73 $$\bar{\bar{M}} = \frac{nk_2 + 4(k_2\gamma_2 - k_1\gamma_1)}{\frac{n^2}{4} + n(\gamma_2+\gamma_3) + 4(\gamma_2\gamma_3 - \gamma_1^2)} \bar{w}a^2$$

The analysis has to be augmented in this case by considering the action of a sidesway force

TABLE 5

Overturning Moment, M_o. Hinged Columns

OCTAGONAL RING

Bending Moments	Twisting Moments
$M_A^1 = -\eta_2 M_o$	$T_A^1 = -\eta_1 M_o$
$M_1^A = -(\eta_2 - 0.07907) M_o$	$T_1^A = -"-$
$M_1^E = -0.7071 (\eta_1 + \eta_2 - 0.07907) M_o$	$T_1^E = -0.7071 (\eta_1 - \eta_2 + 0.07907) M_o$
$M_E = -0.7071 (\eta_1 + \eta_2 - 0.125) M_o$	$T_E = -"-$
$M_2^E = -0.7071 (\eta_1 + \eta_2 - 0.12196) M_o$	$T_2^E = -"-$
$M_2^B = -(\eta_1 - 0.02145) M_o$	$T_2^B = (\eta_2 - 0.10051) M_o$
$M_B^2 = -\eta_1 M_o$	$T_B^2 = -"-$

COLUMNS

Bending Moments at Column Tops	Bending Moments at Column Bases
$M^{x-x}_{col.A\ top} = 2\eta_1 M_o$	$M^{x-x}_{col.A\ base} = 0$
$M^{y-y}_{col.A\ top} = 0$	$M^{y-y}_{col.A\ base} = 0$
$M^{x-x}_{col.B\ top} = 0$	$M^{x-x}_{col.B\ base} = 0$
$M^{y-y}_{col.B\ top} = -2\eta_1 M_o$	$M^{y-y}_{col.B\ base} = 0$
Vertical Loads	**Horizontal Reactions at Base**
$V_{col.A} = -\frac{0.2071}{a} M_o$	$H^{x-x}_{col.A} = 0$
$V_{col.B} = 0$	$H^{y-y}_{col.A} = \frac{2\eta_1}{h} M_o$
	$H^{x-x}_{col.B} = \frac{2\eta_1}{h} M_o$
	$H^{y-y}_{col.B} = 0$

$$P_o = \frac{3}{h}(\bar{\bar{T}} + \bar{\bar{M}})$$

The resulting final moments are tabulated in Table 6; the functions η_3, η_4 and η_5 incorporated in these moment expressions can be given as:

74 $$\eta_3 = \frac{n^3(k+1)(0.011183k^2+0.05602k+0.05819)+n^2(0.03440k^4+}{n^4(k+1)^2+n^3(k+1)(4.268k^2+14.692k+2.781)+n^2(5.268k^4+}$$

$$+\frac{+0.0378k^3+0.9145k^2+0.7934k+0.10276)+n(0.02347k^5+}{+37.58k^3+74.96k^2+31.69k+2.452)+n(1.6350k^5+}$$

$$+\frac{+0.3448k^4+1.5562k^3+2.491k^2+0.9091k+0.04060)+}{+18.111k^4+65.02k^3+81.93k^2+17.984k+0.7147)+}$$

$$+\frac{+(0.06566k^3+0.6495k^2+1.241k+0.06303)^2}{+(0.3431k^3+2.662k^2+4.583k+0.2277)^2}$$

75 $$\eta_4 = \frac{n^4(0.2997k+0.2833)^2+n^3(0.3877k^3+1.6448k^2+1.4020k+}{\text{SAME DENOMINATOR}}$$

$$+\frac{+0.19195)+n^2(0.4798k^4+3.247k^3+6.049k^2+2.181k+0.14088)+}{\text{SAME DENOMINATOR}}$$

$$+\frac{+n(0.13871k^5+1.4318k^4+4.776k^3+5.528k^2+0.9463k+0.03428)+}{\text{SAME DENOMINATOR}}$$

$$+\frac{+(0.08672k^3+0.5703k^2+0.8549k+0.04079)^2}{\text{SAME DENOMINATOR}}$$

76 $$\eta_5 = \frac{n^3(k+1)+(0.0009199k^2+0.03248k+0.02001)+n^2(0.008595k^4+}{\text{SAME DENOMINATOR}}$$

$$+\frac{+0.17677k^3+0.6014k^2+0.4197k+0.04317)+n(0.016427k^5+0.2864k^4+}{\text{SAME DENOMINATOR}}$$

$$+\frac{+1.3743k^3+2.148k^2+0.5796k+0.2662)+(0.06565k^3+}{+0.6753k^2+1.2752k+0.07023)^2}$$

$$\text{SAME DENOMINATOR}$$

As before, over the quadrant B-C bending moments are opposite and torques equal to those listed in Table 6.

TABLE 6

Overturning Moment, M_o. Fixed Columns

OCTAGONAL RING

Bending Moments	Twisting Moments
$M_A^1 = -\eta_4 M_o$	$T_A^1 = -(\eta_3-\eta_5)M_o$
$M_1^A = -(\eta_4+0.4142\eta_5-0.07907)M_o$	$T_1^A = -\,"\,-$
$M_1^E = -0.7071(\eta_3+\eta_4-0.5858\eta_5-0.07907)M_o$	$T_1^E = -0.7071(\eta_3-\eta_4-1.4142\eta_5+0.07907)M_o$
$M_E = -0.7071(\eta_3+\eta_4-0.125)M_o$	$T_E = -\,"\,-$
$M_2^E = -0.7071(\eta_3+\eta_4+0.5858\eta_5-0.12196)M_o$	$T_2^E = -\,"\,-$
$M_2^B = -(\eta_3-0.4142\eta_5-0.02145)M_o$	$T_2^B = (\eta_4+\eta_5-0.10052)M_o$
$M_B^2 = -\eta_3 M_o$	$T_B^2 = -\,"\,-$

COLUMNS

Bending Moments at Column Tops	Bending Moments at Column Bases
$M^{x-x}_{\text{col.A top}} = 2(\eta_3-\eta_5)M_o$	$M^{x-x}_{\text{col.A base}} = -(\eta_3+0.5\eta_5)M_o$
$M^{y-y}_{\text{col.A top}} = 0$	$M^{y-y}_{\text{col.A base}} = 0$
$M^{x-x}_{\text{col.B top}} = 0$	$M^{x-x}_{\text{col.B base}} = 0$
$M^{y-y}_{\text{col.B top}} = -2\eta_3 M_o$	$M^{y-y}_{\text{col.B base}} = (\eta_3-1.5\eta_5)M_o$
Vertical Loads	**Horizontal Reactions at Base**
$V_{\text{col.A}} = \frac{0.2071}{a}(4\eta_5-1)M_o$	$H^{x-x}_{\text{col.A}} = 0$
$V_{\text{col.B}} = 0$	$H^{y-y}_{\text{col.A}} = (3\eta_3-1.5\eta_5)\frac{M_o}{h}$
	$H^{x-x}_{\text{col.B}} = (3\eta_3-1.5\eta_5)\frac{M_o}{h}$
	$H^{y-y}_{\text{col.B}} = 0$

NUMERICAL EXAMPLE

An octagonal frame is to carry a refinery tower of 21.6' diameter and 391.0 kips operating weight. In order to eliminate local torsion the diameter of the inscribed circle of the octagon is assumed to be 0.975 of the loading circle. The base of the vessel has to be supported at an elevation of 23'-6"; the maximum wind force acting on the superstructure is 25.2 kips and attacks horizontally at an elevation of 36'-6" above the base. Assuming that the columns as well as the octagonal ring would have a cross section of 2 ft. by 2 ft., the question is whether it would be more economical to design the pedestal with a hinged or a fixed base.

With these data, the design dimensions and forces can be listed as follows:

$a = 4.367$ ft., $b = c = d = 2.0$ ft., $h = 22.5$ ft.
$w = 6.196$ k/ft. (distributed weight of supported equipment plus dead load)
$P = 25.2$ kips, $M = 945$ ft-kips.
$k = 1.341$, $n = 5.153$

The auxiliary functions will, in turn, have the following values:

For the hinged-base condition, from Eqs. (a), (c) and (e)

$\beta_1 = 0.02511$ $\gamma_1 = 0.15009$ $\eta_1 = 0.004673$
$\gamma_2 = 0.17302$ $\eta_2 = 0.08605$

For the fixed-base condition, from Eqs. (b), (d) and (f)

$\beta_2 = 0.02822$ $\gamma_3 = 0.06723$ $\eta_3 = 0.009082$
$\gamma_4 = 0.07503$ $\eta_4 = 0.08215$
$\gamma_5 = 0.10863$ $\eta_5 = 0.005734$

The moments and forces obtained with the aid of these auxiliary functions are listed for the hinged and fixed-base condition in Tables 7 and 8, respectively. Based on these tables, the following comparison can be drawn between the design conditions for the different modes of base fixity.

Hinged-Base	Fixed-Base
$M_A^1 = 353.67$ ft kips	$M_A^1 = 294.06$ ft kips
$M_1^A = \begin{cases} 43.01 - " - \\ -48.95 - " - \end{cases}$	$M_1^A = \begin{cases} 23.20 - " - \\ -29.86 - " - \end{cases}$
$M_1^E = \begin{cases} 71.47 - " - \\ -79.87 - " - \end{cases}$	$M_1^E = \begin{cases} 35.98 - " - \\ -45.42 - " - \end{cases}$
$M_E = -76.99 - " -$	$M_E = -83.22 - " -$
$T_A^1 = T_1^A = 64.04$ ft kips	$T_A^1 = T_1^A = 31.14$ ft kips
$T_1^E = T_E = 10.66 - " -$	$T_1^E = T_E = 5.97 - " -$
$M_{\text{col. top}} = \begin{cases} 167.29 \text{ ft kips} \\ -155.43 - " - \end{cases}$	$M_{\text{col. top}} = \begin{cases} 65.73 \text{ ft kips} \\ -52.39 - " - \end{cases}$
$M_{\text{col. base}} = 0$	$M_{\text{col. base}} = \begin{cases} 83.69 - " - \\ -90.35 - " - \end{cases}$

$V_{col.}$ = 179.94 kips $V_{col.}$ = 163.71 kips

$H_{col.}$ = 7.43 - " - $H_{col.}$ = 6.94 - " -

This comparison shows that most of the bending moments, and all of the torques (which often govern design in reinforced concrete structures), are materially smaller for the fixed-base condition.

MODIFIED ANALYSIS

The moment expressions and especially the functions involved in the extended analysis of horizontal shear and overturning moment are extremely complex and require a great deal of calculations. Material simplifications in these expressions can be achieved by adopting the simplifying assumption that under the horizontal shear loading (P) identical horizontal reactions (1/4P) are developed at the base of each column. While this assumption will not strictly satisfy statics, it will lead, as shown below, to results in excellent agreement with the extended analysis. In addition to this fact, the significant curtailment of arithmetic involved, and the observation that wind loads cannot be predicted with absolute accuracy, make the use of the modified analysis justified in all cases.

Assuming $H_{col.} \doteq P/4$ for every column, the functions involved in Table 3 for the hinged-base columns simplify to:

77 $$\gamma_1 = 0.125$$

78 $$\gamma_2 = \frac{0.2393k + 0.08839}{k+1}$$

(Note that these equations will satisfy the extended analysis only when k = 0.12886 or k = -3.339, which is a physically meaningless condition; Eq. (7) shows that in actual cases k > 0.6702). Denoting the second of Eqs. (c') by $\bar{\gamma}_2$, the simplifying assumption will transform the expressions given in Table 3 into those shown in Table 9.

For the frame with the fixed-base columns, the assumption of equal horizontal reactions reduces the auxiliary functions involved in the extended analysis to the following expressions:

79 $$\bar{\gamma}_3 = \frac{\gamma_5}{2} = \frac{0.06961\,n(k+1) + 0.014128(0.2538k^2 + 1.2473k + 0.2718)}{n(k+1) + 0.2538k^2 + 1.2473k + 0.2718}$$

80 $$\bar{\gamma}_4 = \frac{n(0.11973k + 0.04423)}{\text{SAME DENOMINATOR}}$$

TABLE 7

Numerical Example 1; on Hinged Columns

OCTAGONAL RING

Location & Direction	Moments or Forces Due to: Vert. Load, w	Horiz. Force, P	Moment, M	Total	Location & Direction	Moments or Forces Due to: Vert. Load, w	Horiz. Force, P	Moment, M	Total
Bending Moments in ft.-kips					**Twisting Moments in ft.-kips**				
M^1_A	+174.26	$\overset{-}{\downarrow}$ 98.10	$\overset{-}{\downarrow}$81.31	+353.67 - 5.15	T^1_A	-2.97	$\overset{-}{\downarrow}$56.65	$\overset{-}{\downarrow}$ 4.42	-64.04
M^A_1	- 2.97	$\overset{-}{\downarrow}$ 39.39	$\overset{-}{\downarrow}$ 6.59	+ 43.01 - 48.95	T^A_1	-2.97	$\overset{-}{\downarrow}$56.65	$\overset{-}{\downarrow}$ 4.42	-64.04
M^E_1	- 4.20	$\overset{-}{\downarrow}$ 67.89	$\overset{-}{\downarrow}$ 7.78	+ 71.47 - 79.87	T^E_1	0	$\overset{-}{\downarrow}$12.20	$\overset{+}{-}$ 1.54	$\overset{+}{-}$10.66
M_E	- 63.27	$\overset{-}{\downarrow}$ 9.19	$\overset{+}{-}$22.91	- 76.99	T_E	0	$\overset{-}{\downarrow}$12.20	$\overset{+}{-}$ 1.54	$\overset{+}{-}$10.66
M^E_2	- 4.20	$\overset{+}{-}$ 49.50	$\overset{+}{-}$20.90	+ 66.20 - 74.60	T^E_2	0	$\overset{-}{\downarrow}$12.20	$\overset{+}{-}$ 1.54	$\overset{+}{-}$10.66
M^B_2	- 2.97	$\overset{+}{-}$ 26.39	$\overset{+}{-}$15.85	+ 39.27 - 45.21	T^B_2	+2.97	$\overset{-}{\downarrow}$43.65	$\overset{-}{\downarrow}$13.66	+60.28
M^2_B	+174.26	$\overset{+}{-}$ 85.10	$\overset{-}{\downarrow}$ 4.42	+254.94	T^2_B	+2.97	$\overset{-}{\downarrow}$43.65	$\overset{-}{\downarrow}$13.66	+60.28

COLUMNS

Bending Moments at Column Tops, ft.-kips					Bending Moments at Column Bases, ft.-kips				
$M^{x-x}_{col.A\ top}$	+ 5.93	$\overset{+}{-}$113.30	$\overset{+}{-}$ 8.84	+128.07 -116.21	$M^{x-x}_{col.A\ base}$	0	0	0	0
$M^{y-y}_{col.A\ top}$	0	0	0	0	$M^{y-y}_{col.A\ base}$	0	0	0	0
$M^{x-x}_{col.B\ top}$	+ 5.93	0	0	+ 5.93	$M^{x-x}_{col.B\ base}$	0	0	0	0
$M^{y-y}_{col.B\ top}$	0	$\overset{+}{-}$170.20	$\overset{-}{\downarrow}$ 8.84	$\overset{+}{-}$161.36	$M^{y-y}_{col.B\ base}$	0	0	0	0
Vertical Loads, kips					**Horizontal Reactions at Base, kips**				
$V_{col.A}$	+108.23	$\overset{-}{\downarrow}$ 26.89	$\overset{-}{\downarrow}$ 44.82	+179.94	$H^{x-x}_{col.A}$	0	0	0	0
$V_{col.B}$	+108.23	0	0	+108.23	$H^{y-y}_{col.A}$	+0.26	$\overset{+}{-}$ 5.04	$\overset{+}{-}$ 0.39	+ 5.69 - 5.17
					$H^{x-x}_{col.B}$	0	$\overset{-}{\downarrow}$ 7.56	$\overset{+}{-}$ 0.39	$\overset{+}{-}$ 7.17
					$H^{y-y}_{col.B}$	+0.26	0	0	+ 0.26

TABLE 8

Numerical Example 2; on Fixed Columns

OCTAGONAL RING

Location & Direction	Moments or Forces Due to: Vert. Load, w	Horiz. Force, P	Moment, M	Total	Location & Direction	Moments or Forces Due to: Vert. Load, w	Horiz. Force, P	Moment, M	Total
Bending Moments in ft.-kips					**Twisting Moments in ft.-kips**				
M^{1}_{A}	+173.89	↓ 42.54	↓ 77.63	+294.06	T^{1}_{A}	-3.33	↓ 23.48	↓ 3.16	-29.97
M^{A}_{1}	- 3.33	↓ 17.03	↓ 5.15	↑ 18.85 - 25.51	T^{A}_{1}	-3.33	↓ 23.48	↓ 3.16	-29.97
M^{E}_{1}	- 4.72	↓ 28.64	↓ 5.88	↑ 29.80 - 39.24	T^{E}_{1}	0	↓ 4.56	↑ 1.41	↑ 5.97
M_{E}	- 63.79	↓ 3.13	↑ 22.56	- 83.22	T_{E}	0	↓ 4.56	↑ 1.41	↑ 5.97
M^{E}_{2}	- 4.72	↑ 22.38	↑ 18.32	+ 35.98 - 45.42	T^{E}_{2}	0	↓ 4.56	↑ 1.41	↑ 5.97
M^{B}_{2}	- 3.33	↑ 12.60	↑ 13.93	+ 23.20 - 29.86	T^{B}_{2}	+3.33	↓ 19.05	↓ 8.76	+31.14
M^{2}_{B}	+173.89	↑ 38.12	↓ 8.58	+203.43	T^{2}_{B}	+3.33	↓ 19.05	↓ 8.76	+31.14

COLUMNS

Bending Moments at Column tops, ft.-kips					Bending Moments at Column Bases, ft.-kips				
$M^{x-x}_{col.A\ top}$	6.67	↑ 46.95	↑ 6.33	↑ 59.95 - 46.61	$M^{x-x}_{col.A\ base}$	-3.33	↓ 72.84	↓ 11.29	+80.80 -87.46
$M^{y-y}_{col.A\ top}$	0	0	0	0	$M^{y-y}_{col.A\ base}$	0	0	0	0
$M^{x-x}_{col.B\ top}$	6.67	0	0	+ 6.67	$M^{x-x}_{col.B\ base}$	-3.33	0	0	- 3.33
$M^{y-y}_{col.B\ top}$	0	↑ 76.23	↓ 17.17	↑ 59.06	$M^{y-y}_{col.B\ base}$	0	↓ 87.48	↑ 0.46	↑ 87.02
Vertical Loads, kips					**Horizontal Reactions at Base, ft.-kips**				
$V_{col.A}$	+108.23	↓ 11.69	↓ 43.79	+163.71	$H^{x-x}_{col.A}$	0	0	0	0
$V_{col.B}$	+108.23	0	0	+108.23	$H^{y-y}_{col.A}$	+0.44	↑ 5.32	↑ 0.78	+ 6.54 - 5.66
					$H^{x-x}_{col.B}$	0	↓ 7.28	↑ 0.78	↑ 6.50
					$H^{y-y}_{col.B}$	+0.44	0	0	+ 0.44

TABLE 9

Modified Analysis
Concentrated Horizontal Load, P . Hinged Columns.

OCTAGONAL RING	
Bending Moments	**Twisting Moments**
$M^1_A = -\bar{\gamma}_2 Ph$	$T^1_A = -0.125\ Ph$
$M^A_1 = -(\bar{\gamma}_2 - 0.10355)\ Ph$	$T^A_1 = -"-$
$M^E_1 = -0.7071\ (\bar{\gamma}_2 + 0.02145)\ Ph$	$T^E_1 = 0.7071\ (\bar{\gamma}_2 - 0.2286)\ Ph$
$M_E = -0.7071\ (\bar{\gamma}_2 - 0.125)\ Ph$	$T_E = -"-$
$M^E_2 = -0.7071\ (\bar{\gamma}_2 - 0.2714)\ Ph$	$T^E_2 = -"-$
$M^B_2 = 0.02145\ Ph$	$T^B_2 = (\bar{\gamma}_2 - 0.25)\ Ph$
$M^2_B = 0.125\ Ph$	$T^2_B = -"-$

COLUMNS	
Bending Moments at Column Tops	**Bending Moments at Column Bases**
$M^{x-x}_{col.A\ top} = 0.25\ Ph$	$M^{x-x}_{col.A\ base} = 0$
$M^{y-y}_{col.A\ top} = 0$	$M^{y-y}_{col.A\ base} = 0$
$M^{x-x}_{col.B\ top} = 0$	$M^{x-x}_{col.B\ base} = 0$
$M^{y-y}_{col.B\ top} = 0.25\ Ph$	$M^{y-y}_{col.B\ base} = 0$
Vertical Loads	**Horizontal Reactions at Base**
$V_{col.A} = -\frac{0.2071}{a} Ph$	$H^{x-x}_{col.A} = 0$
$V_{col.B} = 0$	$H^{y-y}_{col.A} = 0.25\ P$
	$H^{x-x}_{col.B} = -0.25\ P$
	$H^{y-y}_{col.B} = 0$

(Eqs. (d') again involve the physically impossible condition that k = 0.12886 or k = 3.339). Denoting the first and second expression of Eqs. (d') by $\bar{\gamma}_3$ and $\bar{\gamma}_4$, respectively, the moment and force expressions obtained for a fixed-based condition under horizontal thrust using the modified analysis are reproduced in Table 10.

While the simplifying condition does not change the basic steps contained in the overturning moment analysis, the shorter expressions involved in the sidesway consideration lead to materially abbreviated auxiliary functions. In this context, moments and forces for the hinged-base frame can be expressed in terms of the functions

81 $$\bar{\eta}_1 = \frac{0.00805k^2 + 0.02979k + 0.03613}{n(k+1) + 1.125(k^2 + 3.333k + 1)}$$

82 $$\bar{\eta}_2 = \frac{n(0.01072k + 0.0203) + 0.00805k^2 + 0.04411k + 0.04090}{\text{SAME DENOMINATOR}}$$

whereas, for the fixed-base frame, the auxiliary functions are reduced to

83 $$\bar{\eta}_3 = \frac{0.01070k^2 + 0.0397k + 0.04815}{n(k+1) + 1.5(k^2 + 3.333k + 1)}$$

84 $$\bar{\eta}_4 = \frac{n(0.08979k + 0.08072) + 0.1400k^2 + 0.4434k + 0.0965}{\text{SAME DENOMINATOR}}$$

The force and moment expressions based on the modified approach for the frame under an overturning moment loading are given in Tables 11 and 12 for the hinged- and fixed-base condition, respectively. These tables are similar to Table 5, if allowance is made for the different functions encountered.

To compare the order of magnitude entailed in the modified approach, the octagonal frame solved in the previous example was reanalyzed by means of the method. These values are listed in Tables 13 and 14, for the hinged and fixed- base frames, respectively, where the appropriate values derived from the extended analysis (Table 7 and 8) are also shown for comparative purposes. Moments and forces due to distributed vertical loads were not tabulated, since these remain identical by either approach.

Tables 13 and 14 show that the modified theory is in excellent over-all agreement with the extended approach. Insignificant differences occur only at points of low moment values, which from a practical standpoint, are negligible, since they do not govern design.

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TABLE 10
Modified Analysis

Concentrated Horizontal Load, P. Fixed Columns.

OCTAGONAL RING	
Bending Moments	**Twisting Moments**
$M_A^1 = -\bar{\gamma}_4$ Ph	$T_A^1 = -\bar{\gamma}_3$ Ph
$M_1^A = (0.8284\,\bar{\gamma}_3 - \bar{\gamma}_4)$ Ph	$T_1^A =$ - " -
$M_1^E = -0.7071\,(0.17157\,\bar{\gamma}_3 + \bar{\gamma}_4)$ Ph	$T_1^E = -0.7071\,(1.8284\,\bar{\gamma}_3 - \bar{\gamma}_4)$ Ph
$M_E = 0.7071\,(\bar{\gamma}_3 - \bar{\gamma}_4)$ Ph	$T_E =$ - " -
$M_2^E = 0.7071\,(2.172\,\bar{\gamma}_3 - \bar{\gamma}_4)$ Ph	$T_2^E =$ - " -
$M_2^B = 0.17157\,\bar{\gamma}_3$ Ph	$T_2^B = -(2\bar{\gamma}_3 - \bar{\gamma}_4)$ Ph
$M_B^2 = \bar{\gamma}_3$ Ph	$T_B^2 =$ - " -

COLUMNS	
Bending Moments at Column Tops	**Bending Moments at Column Bases**
$M^{x-x}_{col.A\ top} = 2\,\bar{\gamma}_3$ Ph	$M^{x-x}_{col.A\ base} = (2\,\bar{\gamma}_3 - 0.25)$ Ph
$M^{y-y}_{col.A\ base} = 0$	$M^{y-y}_{col.A\ base} = 0$
$M^{x-x}_{col.B\ top} = 0$	$M^{x-x}_{col.B\ base} = 0$
$M^{y-y}_{col.B\ base} = 2\,\bar{\gamma}_3$ Ph	$M^{y-y}_{col.B\ base} = (2\,\bar{\gamma}_3 - 0.25)$ Ph
Vertical Loads	**Horizontal Reactions at Base**
$V_{col.A} = -\frac{1.6569}{a}\,\bar{\gamma}_3$ Ph	$H^{x-x}_{col.A} = 0$
$V_{col.B} = 0$	$H^{y-y}_{col.A} = 0.25$ P
	$H^{x-x}_{col.B} = -0.25$ P
	$H^{y-y}_{col.B} = 0$

TABLE 11
Modified Analysis

Overturning Moment, M_o. Hinged Columns.

OCTAGONAL RING	
Bending Moments	**Twisting Moments**
$M_A^1 = (-0.1005 + \bar{\eta}_2) M_o$	$T_A^1 = -\bar{\eta}_1 M_o$
$M_1^A = (\bar{\eta}_2 - 0.02145) M_o$	$T_1^A = -"-$
$M_1^E = -0.7071 (\bar{\eta}_1 - \bar{\eta}_2 - 0.02145) M_o$	$T_1^E = -0.7071 (\bar{\eta}_1 + \bar{\eta}_2 - 0.02145) M_o$
$M_E = -0.7071 (\bar{\eta}_1 + \bar{\eta}_2 - 0.02448) M_o$	$T_E = -"-$
$M_2^E = -0.7071 (\bar{\eta}_1 + \bar{\eta}_2 - 0.02145) M_o$	$T_2^E = -"-$
$M_2^B = -(\bar{\eta}_1 - 0.02145) M_o$	$T_2^B = -\bar{\eta}_2 M_o$
$M_B^2 = -\bar{\eta}_1 M_o$	$T_B^2 = -"-$

COLUMNS	
Bending Moments at Column Tops	**Bending Moments at Column Bases**
$M^{x-x}_{col.A\ top} = 2\bar{\eta}_1 M_o$	$M^{x-x}_{col.A\ base} = 0$
$M^{y-y}_{col.A\ top} = 0$	$M^{y-y}_{col.A\ base} = 0$
$M^{x-x}_{col.B\ top} = 0$	$M^{x-x}_{col.B\ base} = 0$
$M^{y-y}_{col.B\ top} = 2\bar{\eta}_1 M_o$	$M^{y-y}_{col.B\ base} = 0$
Vertical Loads	**Horizontal Reactions at Base**
$V_{col.A} = -\frac{0.2071}{a} M_o$	$H^{x-x}_{col.A} = 0$
$V_{col.B} = 0$	$H^{y-y}_{col.A} = \frac{2\bar{\eta}_1}{h} M_o$
	$H^{x-x}_{col.B} = \frac{2\bar{\eta}_1}{h} M_o$
	$H^{y-y}_{col.B} = 0$

TABLE 12

Modified Analysis
Overturning Moment, M_o. Fixed Columns.

OCTAGONAL RING	
Bending Moments	**Twisting Moments**
$M_A^1 = -\bar{\eta}_4 M_o$	$T_A^1 = -\bar{\eta}_3 M_o$
$M_1^A = -(\bar{\eta}_4 - 0.07907) M_o$	$T_1^A =$ - " -
$M_1^E = -0.7071 (\bar{\eta}_3 + \bar{\eta}_4 - 0.07907) M_o$	$T_1^E = -0.7071 (\bar{\eta}_3 - \bar{\eta}_4 + 0.07907) M_o$
$M_E = -0.7071 (\bar{\eta}_3 + \bar{\eta}_4 - 0.125) M_o$	$T_E =$ - " -
$M_2^E = -0.7071 (\bar{\eta}_3 + \bar{\eta}_4 - 0.12196) M_o$	$T_2^E =$ - " -
$M_2^B = -(\bar{\eta}_3 - 0.02145) M_o$	$T_2^B = (\bar{\eta}_4 - 0.10051) M_o$
$M_B^2 = -\bar{\eta}_3 M_o$	$T_B^2 =$ - " -

COLUMNS	
Bending Moments at Column Tops	**Bending Moments at Column Bases**
$M^{x-x}_{\text{col.A top}} = 2\bar{\eta}_3 M_o$	$M^{x-x}_{\text{col.A base}} = -\bar{\eta}_3 M_o$
$M^{y-y}_{\text{col.A top}} = 0$	$M^{y-y}_{\text{col.A base}} = 0$
$M^{x-x}_{\text{col.B top}} = 0$	$M^{x-x}_{\text{col.B base}} = 0$
$M^{y-y}_{\text{col.B top}} = -2\bar{\eta}_3 M_o$	$M^{y-y}_{\text{col.B base}} = \bar{\eta}_3 M_o$
Vertical Loads	**Horizontal Reactions at Base**
$V_{\text{col.A}} = -\frac{0.2071}{a} M_o$	$H^{x-x}_{\text{col.A}} = 0$
$V_{\text{col.B}} = 0$	$H^{y-y}_{\text{col.A}} = \frac{3\bar{\eta}_3}{h} M_o$
	$H^{x-x}_{\text{col.B}} = \frac{3\bar{\eta}_3}{h} M_o$
	$H^{y-y}_{\text{col.B}} = 0$

TABLE 13

Comparison between Extended and Modified Analysis

Hinged Columns.

OCTAGONAL RING									
Direction and Location	Moments or Forces Due to Horiz. Force, P		Moments or Forces Due to Moment, M		Direction and Location	Moments or Forces Due to Horiz. Force, P		Moments or Forces Due to Moment, M	
	Exact	Simplified	Exact	Simplified		Exact	Simplified	Exact	Simplified
Bending Moments in ft.-kips					Twisting Moments in ft.-kips				
M_A^1	-98.10	-99.15	-81.31	-81.29	T_A^1	-56.65	-70.88	- 4.42	- 4.23
M_1^A	-39.39	-40.44	- 6.59	- 6.60	T_1^A	- " -	- " -	- " -	- " -
M_1^E	-67.89	-78.71	- 7.78	- 7.64	T_1^E	-12.20	-21.52	+ 1.54	+1.60
M_E	- 9.19	-19.99	+22.91	+ 23.05	T_E	- " -	- " -	- " -	- " -
M_2^E	+49.50	+38.71	+20.90	+ 21.03	T_2^E	- " -	- " -	- " -	- " -
M_2^B	+26.39	+12.16	+15.85	+ 16.04	T_2^B	-43.65	-42.60	-13.66	-13.69
M_B^2	+85.10	+70.88	- 4.42	- 4.23	T_B^2	- " -	- " -	- " -	- " -
COLUMNS									
Bending Moments at Column Tops, ft.-kips					Bending Moments at Column Bases, ft.-kips				
M^{x-x} col.A top	+113.30	+141.75	+ 8.84	+ 8.45	M^{x-x} col.A base	0	0	0	0
M^{y-y} col.A top	0	0	0	0	M^{y-y} col.A base	0	0	0	0
M^{x-x} col.B top	0	0	0	0	M^{x-x} col.B base	0	0	0	0
M^{y-y} col.B top	+170.20	+141.75	- 8.84	- 8.45	M^{y-y} col.B base	0	0	0	0
Vertical Loads, kips					Horizontal Reactions at Base, kips				
$V_{col.A}$	- 26.89	- 26.89	-44.82	- 44.82	$H^{x-x}_{col.A}$	0	0	0	0
$V_{col.B}$	0	0	0	0	$H^{y-y}_{col.A}$	+ 5.04	+ 6.30	+ 0.39	+ 0.38
					$H^{x-x}_{col.B}$	- 7.56	- 6.30	+ 0.39	+ 0.38
					$H^{y-y}_{col.B}$	0	0	0	0

TABLE 14

Comparison between Extended and Modified Analyses

Fixed Columns.

OCTAGONAL RING									
Direction and Location	Moments or Forces Due to Horiz. Force, P		Moments or Forces Due to Moment, M		Direction and Location	Moments or Forces Due to Horiz. Force, P		Moments or Forces Due to Moment, M	
	Exact	Simplified	Exact	Simplified		Exact	Simplified	Exact	Simplified
Bending Moments in ft.-kips					Twisting Moments in ft.-kips				
M^{1}_{A}	-42.54	-41.37	-77.63	-81.34	T^{1}_{A}	-23.48	-29.58	- 3.16	- 4.96
M^{A}_{1}	-17.03	-16.87	- 5.15	- 6.63	T^{A}_{1}	- " -	- " -	- " -	- " -
M^{E}_{1}	-28.64	-32.84	- 5.88	- 8.20	T^{E}_{1}	- 4.56	- 8.99	+ 1.41	+ 1.18
M_{E}	- 3.13	- 8.34	+22.56	+22.50	T_{E}	- " -	- " -	- " -	- " -
M^{E}_{2}	+22.38	+16.18	+18.32	+20.46	T^{E}_{2}	- " -	- " -	- " -	- " -
M^{B}_{2}	+12.60	+ 5.08	+13.93	+15.31	T^{B}_{2}	-19.05	-17.79	- 8.76	-13.64
M^{2}_{B}	+38.12	+29.58	- 8.58	- 4.96	T^{2}_{B}	- " -	- " -	- 8.76	-13.64
COLUMNS									
Bending Moments at Column Tops, ft.-kips					Bending Moments at Column Bases, ft.-kips				
$M^{x-x}_{col.A\ top}$	+46.95	+59.16	+ 6.33	+ 9.93	$M^{x-x}_{col.A\ base}$	-72.84	-82.59	-11.29	- 4.96
$M^{y-y}_{col.A\ top}$	0	0	0	0	$M^{y-y}_{col.A\ base}$	0	0	0	0
$M^{x-x}_{col.B\ top}$	0	0	0	0	$M^{x-x}_{col.B\ base}$	0	0	0	0
$M^{y-y}_{col.B\ top}$	+76.23	+59.16	-17.17	- 9.93	$M^{y-y}_{col.B\ base}$	-87.48	-82.59	+ 0.46	+ 4.96
Vertical Loads, kips					Horizontal Reactions at Base, ft.-kips				
$V_{col.A}$	-11.69	-11.22	-43.79	-44.82	$H^{x-x}_{col.A}$	0	0	0	0
$V_{col.B}$	0	0	0	0	$H^{y-y}_{col.A}$	+ 5.32	+ 6.30	+ 0.78	+ 0.66
					$H^{x-x}_{col.B}$	- 7.28	- 6.30	+ 0.78	+ 0.66
					$H^{y-y}_{col.B}$	0	0	0	0

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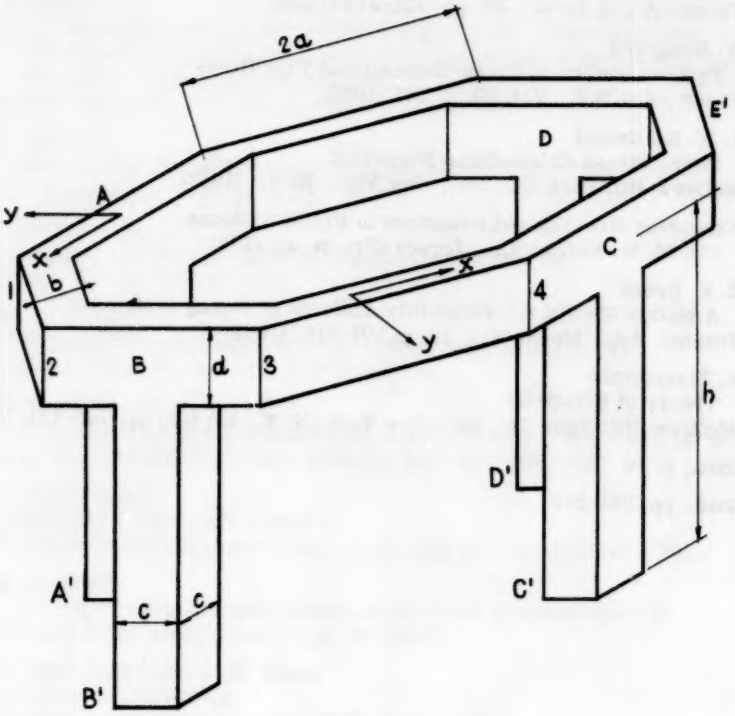

FIG. 1

GENERAL VIEW OF OCTAGONAL FRAME

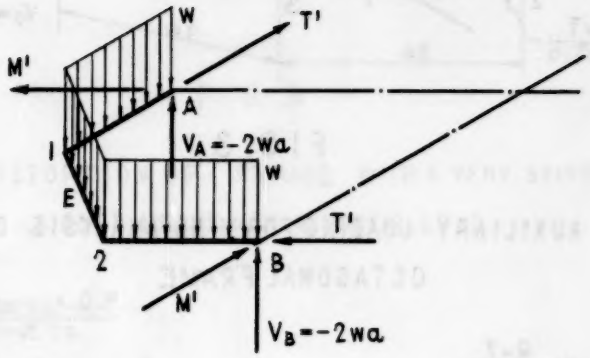

FIG. 2

EQUILIBRIUM CONDITIONS OF OCTAGONAL QUADRANT UNDER A UNIFORMLY DISTRIBUTED VERTICAL LOAD

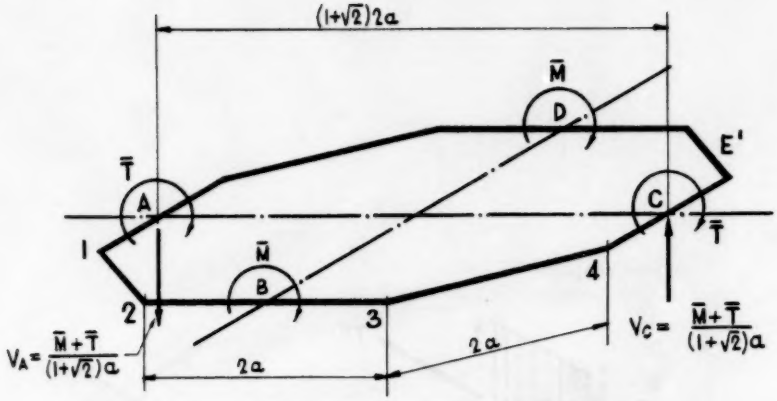

FIG. 3

AUXILIARY LOADING FOR WIND ANALYSIS OF OCTAGONAL FRAME

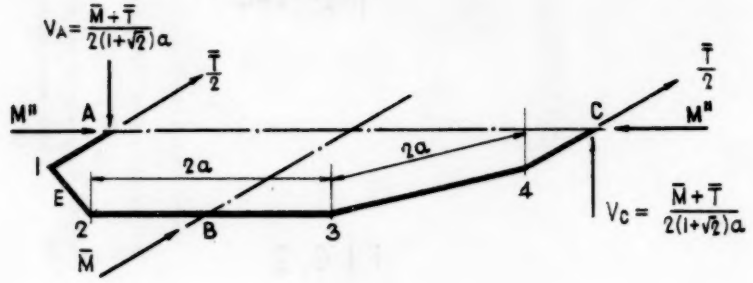

FIG. 4

ANALYSIS OF SEMI-OCTAGONAL RING UNDER THE LOADING SHOWN IN FIG. 3

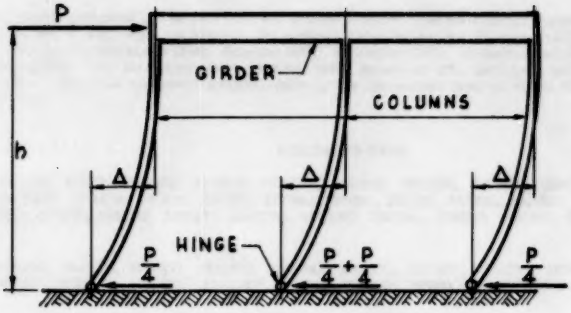

FIG. 5

DISTORTION OF FRAME WITH A VERY STIFF GIRDER UNDER A HORIZONTAL LOAD

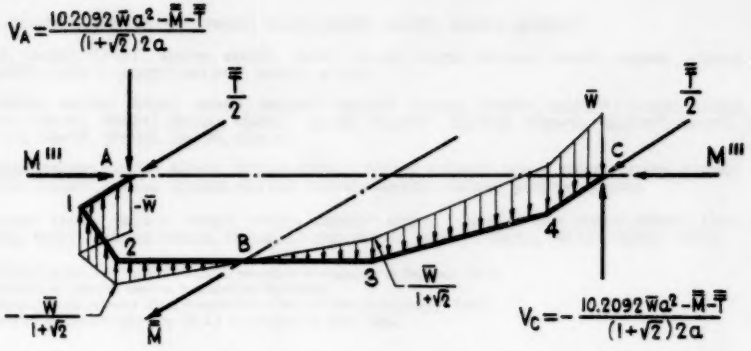

FIG. 6

EXTERNAL AND INTERNAL FORCES FOR THE OVERTURNING MOMENT ANALYSIS OF THE OCTAGONAL FRAME

PROCEEDINGS-SEPARATES

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DECEMBER: 359(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU)[c], 368(WW)[c], 369(IR), 370(AT)[c], 371(SM)[c], 372(CO)[c], 373(ST)[c], 374(EM)[c], 375(EM), 376(EM), 377(SA)[c], 378(PO)[c].

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JANUARY: 379(SM)[c], 380(HY), 381(HY), 382(HY), 383(HY), 384(HY)[c], 385(SM), 386(SM), 387(EM), 388(SA), 389(SU)[c], 390(HY), 391(IR)[c], 392(SA), 393(SU), 394(AT), 395(SA)[c], 396(EM)[c], 397(ST)[c].

FEBRUARY: 398(IR)[d], 399(SA)[d], 400(CO)[d], 401(SM)[c], 402(AT)[d], 403(AT)[d], 404(IR)[d], 405(PO)[d], 406(AT)[d], 407(SU)[d], 408(SU)[d], 409(WW)[d], 410(AT)[d], 411(SA)[d], 412(PO)[d], 413(HY)[d].

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APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

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AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

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NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

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a. Presented at the New York (N.Y.) Convention of the Society in October, 1953.
c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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